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EFFECT OF CHARGE RADIUS ON DETONATION VELOCITY

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EFFECT OF CHARGE RADIUS ON DETONATION VELOCITY

R. B. Parlin and D. W. Robinson

Abstract

The derivation of the detonation velocity of an ideal wave in an imperfect gas obeying the equation of state of the form

$$pv = nRTe^{(K/v)}$$

is given. Assuming the curved-front theory, the effect of radial expansion on the velocity of propagation of a non-ideal detonation wave is treated using the above equation of state. The significant approximations involved are discussed and the final equations are given in a form easily applied to experimental data. The results of this study suggest that longer reaction zone lengths than those previously obtained using a constant co-volume may be appropriate.

EFFECT OF CHARGE RADIUS ON DETONATION VELOCITY

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1. A qualitative discussion of the curved-front theory is given in Chem. Rev. 45, 69 (1949) (Eyring, Duffy, Powell and Parlin) and in Technical Report No. I (January 1, 1952) (Parlin, Thorne and Robinson). Reference for complete details of this theory should be made to these reports. Briefly, however, the theory explains the observed decrease in detonation velocity arising from a reduction in the radius of the charge. Consider a detonation wave propagating along a charge of radius $\underline{R}$ and of infinite length. Due to the expanding gas front, a rarefaction wave is sent into the reaction zone. Such a rarefaction wave will overtake the front of the wave, giving rise to a reduction in its velocity and a curvature of the detonation front.

2. Before proceeding with a general analysis of the curved-front theory, let us derive the detonation velocity of an ideal wave in an imperfect gas, assuming an equation of state of the form

$$pv = nRTe^{x} \tag{2.1}$$

where

$$x = K/v. \tag{2.2}$$

Previous studies have used a constant co-volume equation of state.

This derivation results from a simultaneous solution of the hydrodynamic equations. The equation of continuity for a plane wave is

$$U/D = v/v_o \tag{2.3}$$

and the equation of motion is

$$p + U^2/v = D^2/v_o + p_o \tag{2.4}$$

Furthermore, the equation of energy conservation is

$$\Delta E + pv + 1/2\, U^2 = \Delta Q + 1/2\, D^2 + p_o v_o \tag{2.5}$$

where

$$\Delta E = \overline{C}_v\ (T - T_o) = \overline{C}_v\ (pv/nRe^x - T_o) \tag{2.6}$$

and $\overline{C}_v = \dfrac{\int_{T_o}^{T} C_v dT}{T - T_o}$, since the internal pressure $p_i = T\left(\frac{dp}{dT}\right)_v - p$ is zero by equation (2.1).

Substituting (2.6) into (2.5),

$$L'pv + 1/2\ (U^2 - D^2) = Q \tag{2.7}$$

where

$$Q = \Delta Q + \overline{C}_v T_o \tag{2.8a}$$

and

$$L' = \overline{C}_v/nRe^x + 1 \tag{2.8b}$$

and where p_o is assumed to be negligible compared to p. The Chapman Jouguet condition is

$$D = v_o \sqrt{-\left(\frac{dp}{dv}\right)_S}\ . \tag{2.9}$$

Differentiating (2.1) with respect to v, and using the thermodynamic relations

$$\left(\frac{dT}{dv}\right)_S = -\frac{p}{C_v}\ ,\quad \left(\frac{dx}{dv}\right)_S = -\frac{x}{v}$$

we have

$$\left(\frac{dp}{dv}\right)_S = -\frac{p}{v}\frac{1}{L}$$

where

$$1/L = 1 + nRe^x/C_v + x \tag{2.10}$$

Hence, by (2.9),

$$D^2/v_o^{\ 2} = p/vL \tag{2.11}$$

From (2.3), (2.4) and (2.11)

$$D^2/p = v_o/(1 - v/v_o) = v_o^{\ 2}/vL$$

or

$$v/v_o = 1/(1+L) \qquad (2.12)$$

For the explosive EDNA, the ratio of v/v_o given by (2.12) has been calculated as follows for x in the range 0.8 to 2.2, using $\underline{C_v}$ and $\underline{n}$ data obtained in Technical Report No. VI.

x	0.8	1.0	1.2	1.6	1.6	2.8	2.0	2.2
v/v_o	0.708	0.724	0.745	0.763	0.778	0.792	0.805	0.815

This illustration indicates the degree of error introduced by the approximation

$$v/v_o = 3/4, \qquad (2.13)$$

which will be assumed reasonable for most solid explosives and used in this discussion.

Also, from 2.7, using 2.3, 2.11 and 2.12,

$$(L'L + 1/2)\ (1+L)^{-2} - 1/2 = Q/D_i^{\,2}$$

where D_i is the ideal detonation velocity.

3. Let us know proceed to analyze the non-ideal detonation wave under the assumptions of the curved-front theory. Assume as a first-order approximation that any small region of the wave-front is spherical. Then the normal detonation velocity D may be calculated from the conservation equations appropriate to a spherical detonation wave with r_o the local radius of curvature.

If α is the angle the normal makes with the axis of propagation and if the wave proceeds without a change of shape (steady-state)

$$D = D_o \cos \alpha$$

where D_o is the propagation velocity of the wave itself.

Integration of the differential equation of continuity for a spherical shock leads to

$$U/D = v\,\theta/v_o \qquad (3.1)$$

where

$$\theta = 1 + 2\frac{v_o}{D}\int_{r_o}^{r_o - a_o} \frac{W}{vr} dr. \qquad (3.2)$$

Here D is the normal detonation velocity, W the particle velocity, v the specific volume of the reaction products, r the radial distance coordinate, r_o the radius of curvature, and a_o the (constant) length of the reaction zone. Using (2.13) and assuming the relation $W/D = 1 - v/v_o$ appropriate to a plane wave, by integration of (3.2) we have the approximation

$$\theta = 1 + 2/3 \ln \left(1 - \frac{a_o}{r_o}\right). \qquad (3.3)$$

Using (3.1), the equation of motion for a plane wave in a steady-state can be written

$$p + \frac{(1+\theta^2)}{2\theta^2} \frac{U^2}{v} = \frac{D^2}{v_o}. \qquad (3.4)$$

The equation of energy and the Chapman-Jouguet conditions are given in Section 2. In solving these four hydrodynamic relations for D, let

$$z = pv/D^2, \quad b = v/v_o, \quad \text{and } \omega = U/D$$

then,

$$\omega = b\theta \qquad (3.5)$$

$$z + \frac{1+\theta^2}{2\theta^2}\omega^2 = b \qquad (3.6)$$

$$L'z + 1/2\,(\omega^2 - 1) = Q/D^2 \qquad (3.7)$$

$$b^2 L = z \qquad (3.8)$$

By (3.5), (3.6) and (3.8),

$$b = \frac{2}{2L+(1+\theta^2)} \tag{3.9}$$

If Now we set

$$2\varphi = 1 - \theta^2, \tag{3.10}$$

then (3.9) becomes

$$b = \frac{1}{1+L-\varphi}\ . \tag{3.11}$$

By (3.8) and (3.11),

$$z = \frac{L}{(1+L-\varphi)^2} \tag{3.12}$$

Also,

$$\omega^2 = \frac{1-2\varphi}{(1+L-\varphi)^2} \tag{3.13}$$

Substituting (3.12) and (3.13) into (3.7),

$$(LL' + 1/2\,(1-2\varphi))\ \frac{1}{(1+L-\varphi)^2} - 1/2 = {}^{Q}/_{D^2}\ ,$$

or

$$A + L\varphi - 1/2\,\varphi^2 = (1+L-\varphi)^2\ {}^{Q}/_{D^2}\ , \tag{3.14}$$

where

$$A = L\,(L' - 1 - 1/2\,L). \tag{3.15}$$

Under ideal conditions, $\varphi = 0$, i.e.,

$$A = (1+L)^2\ Q/D_i^{\,2}$$

Hence, from (3.14),

$$1 + \frac{L}{A}\varphi - \frac{1}{2A}\varphi^2 = \frac{(1+L-\varphi)^2}{(1+L)^2} \frac{D_i^2}{D^2} . \tag{3.16}$$

For small values of φ, a good approximation to (3.16) is

$$\frac{D}{D_i} = \frac{1-M'\varphi}{1+N'\varphi} \tag{3.17}$$

where

$$M' = 1/(1+L), \quad N' = \frac{1}{2(L'-1)-L} \tag{3.18}$$

As an example of the percentage error involved in the approximation of equation (3.17), consider the case of $x = 1.8$, $C_v = 0.44$, $\bar{C}_v = 0.38$ and $n = 34.94$ for the composition EDNA. The error is less than 2 per cent for φ less than 0.1, which corresponds to greater than 0.9 (and less, of course, than 1.0).

4. In order to determine the shape of the detonation front, it is sufficient to find a parametric representation of the cross-section of the front, recognizing the symmetry of the wave around the axis of the charge (see Fig. 1). For any point of the front, the radius of curvature is given by $r_o = ds/d\alpha$, where s is the distance measured along the front, and α the angle the normal makes with the wave-front. Also, from (3.3) and (3.10),

$$\varphi = -\,2/3 \ln\left(1 - \frac{a_o}{r_o}\right) \cdot \left(1 + 1/3 \ln\left(1 - \frac{a_o}{r_o}\right)\right)$$

A plot of this function (Fig. 2), gives a good linear approximation in the form

$$\varphi = 0.7\, a_o/r_o \tag{4.1}$$

for $a_o/r_o < 0.75$.

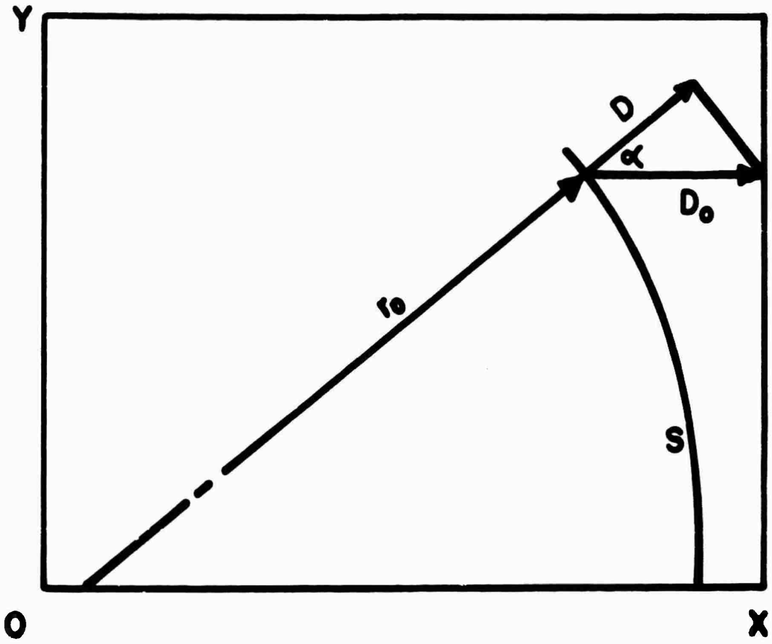

FIG. 1

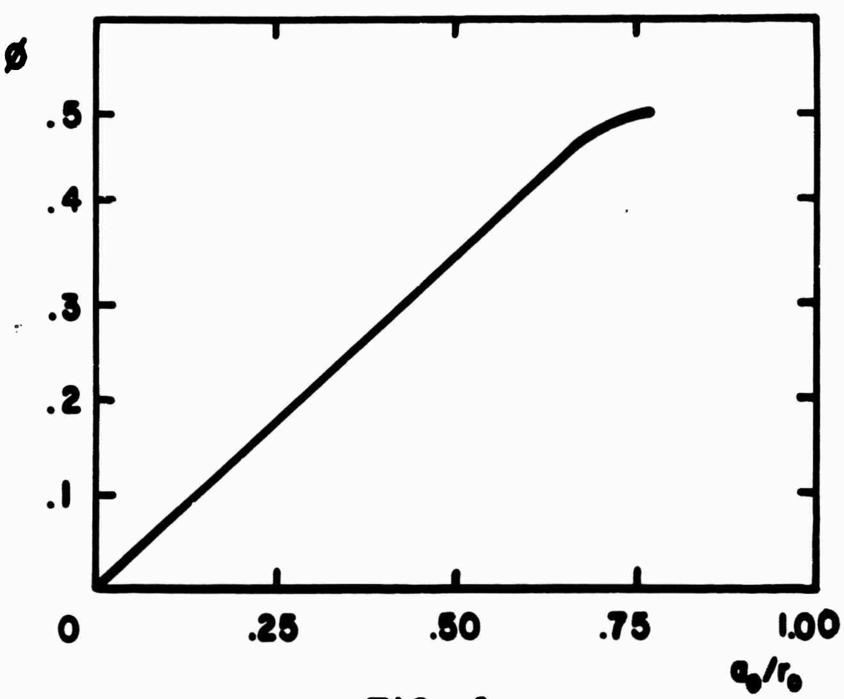

FIG. 2

If $D_o/D_i = \delta$, then since $D = D_o \cos\alpha$, by using Eq. (3.17),

$$D/D_i = \delta\cos\alpha = \frac{1-a_o M \, {}^{d\alpha}/_{dS}}{1+a_o N \, {}^{d\alpha}/_{dS}} \tag{4.2}$$

where $M = 0.7\,M'$, and $N = 0.7N'$. Proceeding as in Technical Report No. 1, we obtain

$$y = a_o((M+N)K_\alpha - N\sin\alpha) \tag{4.3}$$

$$x = -a_o\left(\frac{M+N}{\delta}\ln\frac{1-\delta\cos\alpha}{1-\delta} - \delta(1-\cos\alpha)\right)$$

where

$$K_\alpha = \frac{2}{\delta\sqrt{1-\delta^2}}\tan^{-1}\left(\sqrt{\frac{1+\delta}{1-\delta}}\cdot\tan\frac{\alpha}{2}\right) - \frac{\alpha}{\delta} \tag{4.4}$$

Equations (4.3) give the parametric representation of the wave front. For the case of a solid explosive surrounded by air, the final angle can be taken to the $\pi/2$. For this case we have,

$$y_c/\alpha = (M+N)K_c - N \tag{4.5}$$

where

$$K_c = \frac{2}{\delta\sqrt{1-\delta^2}}\tan^{-1}\sqrt{\frac{1+\delta}{1-\delta}} - \frac{\pi}{2\delta} \tag{4.6}$$

The universal function K_c is given in the following table.

δ	K_c	δ	K_c
0.000	1.00	.850	3.93
.200	1.19	.875	4.44
.300	1.32	.900	5.11
.400	1.45	.925	6.13
.500	1.69	.950	7.86
.600	1.99	.975	11.86
.700	2.45	.990	19.90
.750	2.78	.994	26.32
.800	3.24	.997	38.12

As an illustration, using theoretical data for the composition EDNA as given in Technical Report No. VI, the following results are obtained. Figure 3 is a plot of the ratios D_o/D_i to a_o/y_c for various densities.

ρ_o	δ	0.997	0.994	0.990	0.975	0.950	0.900	0.850	0.750
1.58	a_o/r_o	0.022	0.032	0.042	0.072	0.11	0.18	0.24	0.36
1.20		0.025	0.037	0.049	.084	0.13	.21	0.28	0.42
0.93		0.028	0.041	0.055	.094	0.15	.23	0.31	0.46

The relationship of D_o/D_i to a_o/y_c is often approximated linearly. Such an approximation introduces about a five per cent error for ideal conditions, as can be seen by a linear continuation of the curves in Figure 3.

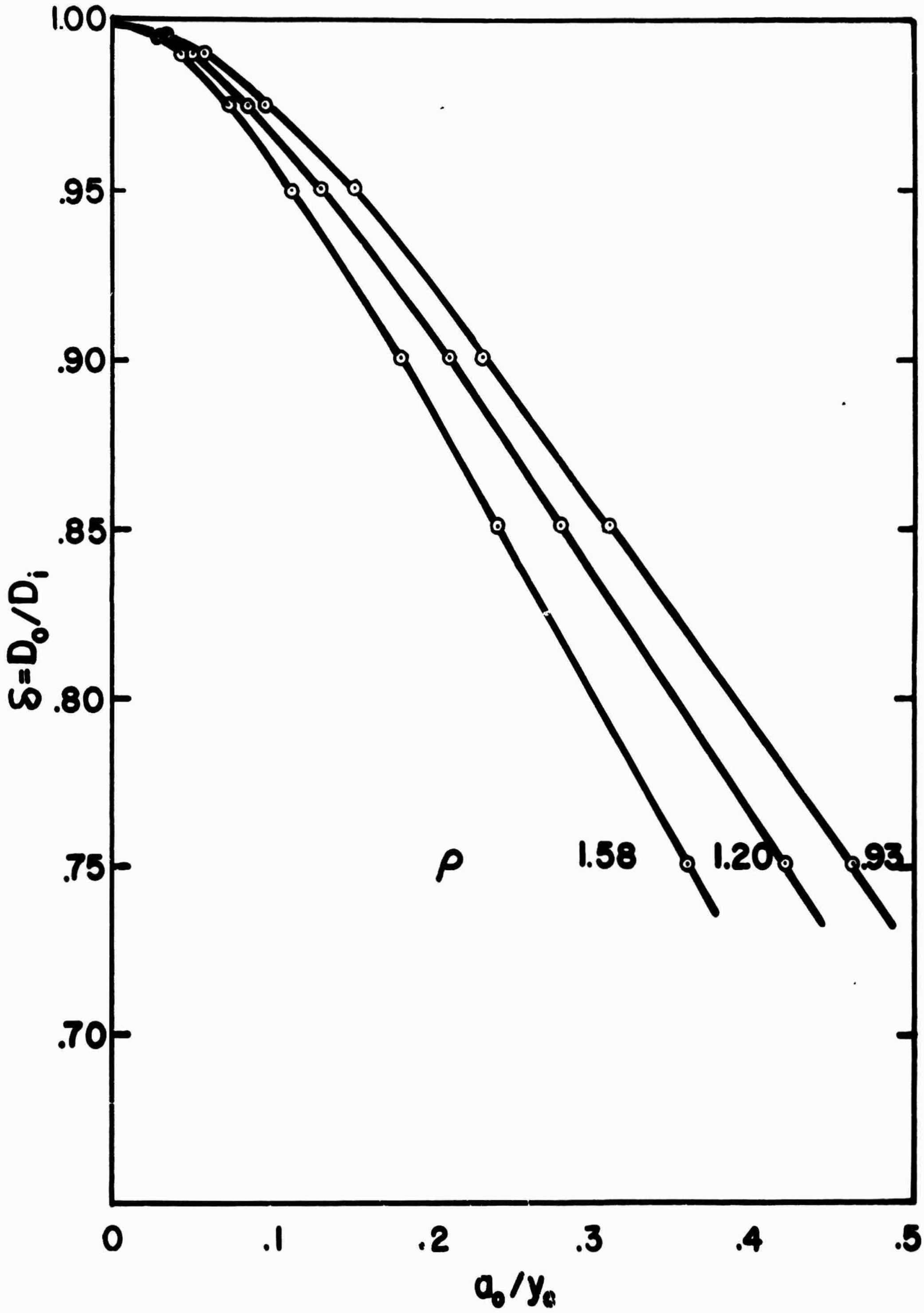

FIG. 3